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When  $\theta=0$ ,  $d\theta/dt=0$ ; hence  $K=2g$ , and we have

$$a\left(\frac{d\theta}{dt}\right)^2 = 2g(1-\cos\theta) + ac^2 \sin^2\theta.$$

Putting  $ac^2=ng$ , and integrating between limits  $\theta=\pi$ ,  $\theta=\frac{1}{2}\pi$ , we find

$$t = \sqrt{\frac{a}{(n+1)g} \log(\sqrt{n+2} + \sqrt{n+1})}.$$

(2) The energy of the particle relative to the tube is  $\frac{1}{2}m a^2 \theta'^2$ . This is due to two causes, gravity and rotation. Hence

$$\frac{1}{2}m a^2 \theta'^2 = amg(1-\cos\theta) + \frac{1}{2}m a^2 \varphi'^2 \sin^2\theta,$$

which is the same as the first integral second equation of the Lagrangian groups.

### AVERAGE AND PROBABILITY.

74. Proposed by F. ANDEREGG, A. M., Professor of Mathematics, Oberlin College, Oberlin, Ohio.

From a point in the circumference of a circular field a projectile is thrown at random with a given velocity which is such that the diameter of the field is equal to the greatest range of the projectile. Find the chance of its falling into the field. [From Byerly's *Integral Calculus*, page 209.]

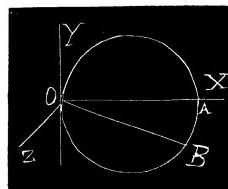
#### I. Solution by the PROPOSER.

It is easily seen that if  $\alpha$ , the angle of projection, has a value from  $\frac{1}{2}\pi - \frac{1}{2}\phi$  to  $\frac{1}{2}\pi + \frac{1}{2}\phi$ , the projectile will fall beyond  $B$ .

The unfavorable chance is

$$\frac{1}{\pi} \int_0^{\frac{1}{2}\pi} \int_{\frac{1}{2}\pi - \frac{1}{2}\phi}^{\frac{1}{2}\pi + \frac{1}{2}\phi} \cos\alpha d\alpha d\phi = \frac{2}{\pi} (\sqrt{2} - 1).$$

Since half of the projectiles will fall on the left of the  $YZ$  plane, the favorable chance is  $\frac{1}{2} - (2/\pi)(\sqrt{2} - 1)$ .



II. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let  $\varphi$ =angle of projection,  $OA=2a$ ,  $\angle AOB=\theta$ .

$\therefore$  Range  $=(v^2/2g)\sin 2\varphi=OB=2a\cos\theta$ , when the projectile falls within the field.

The range is greatest when  $\theta=45^\circ$ , and then according to the conditions of the problem  $v^2/2g=2a$ .

$\therefore 2a\sin 2\varphi=2a\cos\theta$ , or  $\sin 2\varphi=\cos\theta$ .

$\therefore \sin\varphi=\frac{1}{2}(1+\cos\theta)^{\frac{1}{2}} \pm \frac{1}{2}(1-\cos\theta)^{\frac{1}{2}}$ .

Therefore the projectile will fall without the circle, if  $\sin\varphi$  is less than  $\frac{1}{2}(1+\cos\theta)^{\frac{1}{2}} - \frac{1}{2}(1-\cos\theta)^{\frac{1}{2}}$ ; but will fall within if  $\sin\varphi$  is greater than  $\frac{1}{2}(1+\cos\theta)^{\frac{1}{2}} + \frac{1}{2}(1-\cos\theta)^{\frac{1}{2}}$ .

If all possible directions are equally probable, the chance of the projectile falling within the circle is  $1-(1-\cos\theta)^{\frac{1}{2}} = 1-\sqrt{2}\sin^{\frac{1}{2}}\theta$ .

Hence the required chance is

$$p = \frac{\int_0^{\frac{1}{2}\pi} (1-\sqrt{2}\sin^{\frac{1}{2}}\theta)d\theta}{\int_0^{\pi} d\theta} = \frac{1}{2} - (2/\pi)(\sqrt{2}-1).$$

### III. Solution by GEORGE LILLEY, Ph. D., Professor of Mathematics, University of Oregon, Eugene, Ore.

The diameter of the field is  $v^2/g$ , and the range must not exceed  $(v^2/g)\cos\phi$ . The elevation may vary from  $0^\circ$  to  $\theta_1$ , and from  $\frac{1}{2}\pi-\theta_1$  to  $\frac{3}{2}\pi$  for each value of  $\phi$ , where  $\theta_1$  is determined by  $\sin 2\theta_1 = \cos\phi$ ,  $\phi$  the azimuth of the projectile measured from the diameter,  $\theta$  the elevation of the gun,  $v$  the velocity of projection, and  $g$  the intensity of gravity.

The surface-element of the enveloping hemisphere whose radius is  $R$  or  $v^2/g$  is  $R^2\cos\phi d\theta d\phi$ .

Using only one-half the field and one-fourth of the sphere the required chance is

$$\frac{R^2 \int_0^{\frac{1}{2}\pi} \int_0^{\theta_1} \cos\theta d\theta d\phi + R^2 \int_0^{\frac{1}{2}\pi} \int_{\frac{1}{2}\pi-\theta_1}^{\frac{3}{2}\pi} \cos\theta d\theta d\phi}{\pi R^2}$$

$$\text{or } \frac{1}{\pi} \int_0^{\frac{1}{2}\pi} (1 - \sqrt{2}\sin^{\frac{1}{2}}\phi)d\phi, \text{ or } \frac{1}{2} - \frac{2}{\pi}(\sqrt{2}-1).$$

Solved in a similar manner by L. C. WALKER. Solved with different results by HENRY HEATON, and P. H. PHILBRICK.

75. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy, Irving College, Mechanicsburg, Pa.

Find the mean area of all plane rectilineal right triangles having a constant perimeter  $p$ .

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let  $x$  and  $y$  be the base and altitude, respectively.

Then  $x+y+\sqrt{x^2+y^2}=p$ , the perimeter .....(1).

$$\therefore x = \frac{p^2 - 2py}{2(p-y)}. \quad \therefore \text{Area} = \frac{p(p-2y)y}{4(p-y)}.$$